Macro, Money and Finance: A Continuous Time Approach

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Trinity of Stability Conference
Princeton, Nov. 6th, 2015
- Price stability
  Monetary policy

- Financial stability
  Macroprudential policy

- Fiscal debt sustainability
  Fiscal policy

- Short-term interest
- Policy rule (terms structure)

- Reserve requirements
- Capital/liquidity requirements
- Collateral policy
  Margins/haircuts
- Capital controls
Macro, Money and Finance

- **Endogenous level**
  - Persistence & amplification
  - “Net worth trap”

- **Endogenous risk dynamics**
  - Tail risk
  - Crisis probability
  - “Volatility Paradox”

- **Illiquidity and liquidity mismatch**
  - Undercapitalized sectors
  - Time varying risk premia (dynamics)
  - External funding premium

- **Value of money**

- **Welfare**
  - Interaction: regulatory, monetary and other policies
History: Macro & Finance

- **Verbal Reasoning** *(qualitative)*
  - Fisher, Keynes, ...

- **Timeline**
  - **Macro**
    - Growth theory
      - *Dynamic (cts. time)*
      - *Deterministic*
    - Introduce stochastic
      - *Discrete time*
        - Brock-Mirman
        - Kydland-Prescott
        - DSGE models
    - **Finance**
      - Portfolio theory
        - *Static*
        - *Stochastic*
      - Introduce dynamics
        - *Continuous time*
          - Options
          - Term structure
          - Agency theory

- **Cts. time macro with financial frictions**
Amplification & Persistence

  - Perfect (technological) liquidity, but persistence
  - Bad shocks erode net worth, cut back on investments, leading to low productivity & low net worth of in the next period
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  - Technological/market illiquidity
  - KM: Leverage bounded by margins; BGG: Verification cost (CSV)
  - Stronger amplification effects through prices (low net worth reduces leveraged institutions’ demand for assets, lowering prices and further depressing net worth)
Amplification & Persistence

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  - Stronger **amplification** effects through **prices** (low net worth reduces leveraged institutions’ demand for assets, lowering prices and further depressing net worth)

- “once and for all shock”
  - no volatility dynamics
Impulse response vs. Volatility dynamics

- “once and for all shock”
  = no uncertainty about length of slump
- Sequence of adverse shock
Why continuous time modeling?

- Characterization for volatility and amplification
  - Discrete: only impulse response functions
    - Only for shocks starting at the steady state
    - Only expected path – fan charts help somewhat
  - More analytical steps
    - Return equations
      - Next instant returns are essentially log normal
        (easy to take expectations)
    - Explicit net worth and state variable dynamics
      - Continuous: only slope of price function determines amplification
      - Discrete: need whole price function (as jump size can vary)
- Numerically simple – solve differential equations
- Discrete: IES/RA within period = $\infty$, across periods $1/\gamma$
Cts. time: special features of diffusions

- Continuous path – fast enough deleveraging
  - Never jumps over a specific point, e.g. insolvency point

- Implicit assumption: can react to small price changes
  - Can continuously delever as wealth goes down
  - Makes them more bold ex-ante
Recent macro literature (in cts time)

- **Core**

- **Intermediation/shadow banking**

- **Quantification**
  - He & Krishnamurthy (2014), Mittnik & Semmler (2013)

- **International**
  - BruSan (2015), Maggiori (2013)

- **Monetary**
  - “The I Theory of Money” (2012), Drechsler et al. (2014)

- ....
Financial frictions

- Costly state verification (BGG)

- Leverage constraints
  - Exogenous limit (Bewley/Ayagari)

  - Collateral constraints
    - Next period’s price (KM)
      \[ Rb_t \leq q_{t+1} k_t \]
    - Next periods volatility (VaR)
    - Current price

- Incomplete markets
  - Endogenous leverage

\[ \text{Debt limit can depend on prices/volatility} \]
Roadmap

- Why continuous time?
- Literature

Simple model
- With undesirable features
- Add portfolio choice with general utility function

Full model
- With all desired features
- Add equity issuance
A simple model

Basak & Cuoco (1998)

**Experts**
- Output: $y_t = ak_t$
- Consumption rate: $c_t$
- Investment rate: $\lambda_t$

\[
\frac{dk_t}{k_t} = (\Phi(\lambda_t) - \delta) dt + \sigma dZ_t
\]

\[E_0\left[ \int_0^\infty e^{-\rho t} \log(c_t) \, dt \right]\]

- Can only issue risk-free debt

**Households**
- No output: $a = 0$
- Consumption rate: $c_t$

\[E_0[\int_0^\infty e^{-\rho t} \log(c_t) \, dt]\]
Equilibrium

- An equilibrium consists of functions that for each history of macro shocks \( \{Z_s, s \in [0, t]\} \) specify
  - \( q_t \) the price of capital
  - \( k_t, k_t = 0 \) capital holdings
  - \( c_t \geq 0, c_t = 0 \) consumption of representative expert and households
  - \( \iota_t, \iota_t = 0 \) rate of internal investment, per unit of capital
  - \( r \) the risk-free rate

- such that
  - intermediaries and households maximize their utility, taking prices \( q_t \) as given and
  - markets for capital and consumption goods clear
Equilibrium

- Equilibrium is a **map**
  
  Histories of shocks \( \{Z_s, s \leq t\} \)

  \[
  \eta_t = \frac{N_t}{q_t K_t} \in (0,1)
  \]

  **wealth distribution**

  Experts’ wealth share

- Experts, HH solve optimal investment, portfolio, consumption

- Markets clear
Solution steps

1. Postulate endogenous processes
   • \[ \frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^q dZ_t \]
     - Returns from holding capital

2. Equilibrium conditions
   • Agents’ optimization
     - Internal investment (new capital formation)
     - Optimal portfolio choice
     - Optimal consumption
   • Market clearing conditions

3. Law of motion of state variable
   • wealth (share) distribution \( \eta_t \)

4. Express in ODEs of state variable
1. Postulate endogenous process

Postulate

\[ dq_t/q_t = \mu_t^q dt + \sigma_t^q dZ_t \]

Return on capital

\[ dr_t^k = \frac{a - \iota_t}{q_t} dt + \frac{d(k_tq_t)}{k_tq_t} \]

\[ \text{dividend yield} \]
\[ \text{capital gains} \]

\[ d(k_tq_t) = (\Phi(\iota_t) - \delta + \mu_t^q + \sigma_t^q) dt + (\sigma + \sigma_t^q) dZ_t \]

by Ito’s product rule

Recall

\[ dk_t/k_t = (\Phi(\iota_t) - \delta) + \sigma dZ_t \]

In this simple model it will turn out that \( q \) is constant, i.e. \( \mu_t^q = \sigma_t^q = 0 \).
2. Equilibrium optimality conditions

a. Investment rate (capital formation)
   - Static problem
     \[ \max_t \Phi(\lambda_t) - \lambda_t/q_t \]
   - FOC: \( \Phi'(\lambda_t) = \frac{1}{q_t} \) (marginal Tobin’s q)

b. Consumption choice
   - \( c_t = \rho N_t \) due to log utility

c. Portfolio choice
   - Volatility of wealth = Sharpe ratio of risky investment
2. Equilibrium market clearing conditions

- **Goods market price of capital**

\[ \rho q_t K_t = (a - \iota_t(q_t))K_t \]

- \( q_t = q = \ldots \)
- Special case: \( \Phi(t) = \frac{\log(\kappa \iota + 1)}{\kappa} \) \[ \iota = \frac{(q-1)}{\kappa} \]

\[ q = \frac{a+1/\kappa}{r+1/\kappa} \]

- **Risk free rate**

\[ dr_t^k = \frac{a-\iota_t}{q_t} + \left( \Phi(\iota_t) - \delta \right)dt + \sigma dZ_t \]

- Sharpe ratio:
- Volatility of net worth:
- Sharpe ratio = volatility of \( N_t \)

\[ r_t = \rho + \Phi(\iota) - \delta - \frac{\sigma^2}{\eta_t} \]
3. Law of motion of $\eta_t$

- \[ \frac{dN_t}{N_t} = r_t \, dt + \frac{\sigma}{\eta_t} \, dt + \frac{\sigma}{\eta_t} \, dZ_t - \rho \, dt \]

- \[ d\left(\frac{q_tK_t}{q_tK_t}\right) = . . \]

- Use Ito ratio rule for $\eta_t = N_t/(q_tK_t)$

\[ \frac{d\eta_t}{\eta_t} = \frac{(1-\eta_t)^2}{\eta_t^2} \sigma^2 \, dt + (1-\eta_t) \sigma \, dZ_t \]
Observations

\[ \frac{d\eta_t}{\eta_t} = \frac{(1-\eta_t)^2}{\eta_t^2}\sigma^2 dt + (1-\eta_t)\sigma dZ_t \]

- Wealth share \( \eta \) moves with macro shock \( dZ_t \)
- In the long run experts “save their way out”, \( \eta \rightarrow 1 \)

- Sharpe ratio \( \frac{\rho + \Phi(\nu) - \delta - r_t}{\sigma} \)
  - Increases as \( \eta \) goes down, (to \( \infty \) as \( \eta \rightarrow 0 \))
  - Achieved through a lower risk free rate

- \( q \) is constant
  - No endogenous risk
  - No amplification
  - No volatility effects
Generalizing preference: portfolio choice

1. Also postulate process for marginal utility
   \[ d\theta_t / \theta_t = \mu_t^\theta dt + \sigma_t^\theta dZ_t \]
   SDF: \( e^{\rho_s \theta_{t+s} / \theta_t} \)

2. Portfolio choice: Optimality condition
   - For asset \( A \) with payoff process \( dr_t^A = \mu_t^A dt + \sigma_t^A dZ_t \)
     \[ 0 = \mu_t^\theta - \rho + \mu_t^A + \sigma_t^A \sigma_t^\theta \]
   - Intuition:
     i. Discrete time analog: Take log of \( 1 = E_t [SDF_{t,t+s}(R_{t,t+s})] \)
     ii. Consider wealth \( n_t \) invested in \( A \), so that \( dn_t / n_t = dr_t^A \)
        \( n_{t+s} e^{-\rho_s \theta_{t+s} / \theta_t} \) is a martingale
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   • For risk free asset
     \[ 0 = \mu^\theta_t - \rho + r \]
   • Sharpe ratio
     \[ \frac{\mu^A_t - r_t}{\sigma^A_t} = -\sigma^\theta_t \]
1. Also postulate process for marginal utility
   \[ d\theta_t/\theta_t = \mu_t^{\theta} dt + \sigma_t^{\theta} dZ_t \]
   SDF: \( e^{\rho s \theta_{t+s}/\theta_t} \)

2. Portfolio choice: Optimality condition
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     \[ dr_t^A = \mu_t^A dt + \sigma_t^A dZ_t \]
   \[
   0 = \mu_t^\theta - \rho + \mu_t^A + \sigma_t^A \sigma_t^{\theta}
   \]
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     ii. Consider wealth \( n_t \) invested in \( A \), so that
        \[ dn_t/n_t = dr_t^A \]
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     0 = \mu_t^\theta - \rho + r
     \]
   - Sharpe ratio
     \[
     \frac{\mu_t^A - r_t}{\sigma_t^A} = -\sigma_t^{\theta}
     \]

Example 1: \( u(c) = \log(c) \)
\[
\theta_t = \frac{1}{c_t} = \frac{1}{\rho n_t} \quad \Rightarrow \quad \sigma_t^{\theta} = -\sigma_t^n
\]
Generalizing preference: portfolio choice

1. Also postulate process for marginal utility
   \[ d\theta_t / \theta_t = \mu^\theta_t \, dt + \sigma^\theta_t \, dZ_t \]
   SDF: \( e^{\rho_s \theta_{t+s} / \theta_t} \)

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     \[ 0 = \mu^\theta_t - \rho + r \]
   - Sharpe ratio
     \[ \frac{\mu^A_t - r_t}{\sigma^A_t} = -\sigma^\theta_t \]

Example 1: \( u(c) = \log(c) \)
\[ \theta_t = \frac{1}{c_t} = \frac{1}{\rho n_t} \Rightarrow \sigma^\theta_t = -\sigma^n_t \]

Example 2: \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \)
\[ \Rightarrow \sigma^\theta_t = -\gamma \sigma^c_t \]
Desired model properties

- Normal regime: stable around steady state
  - Experts are adequately capitalized
  - Experts can absorb macro shock
- Net worth trap  look at stationary distribution
- Endogenous risk
  - Fat tails
  - Assets are more correlated
  - SDF vs. cash-flow news
- Volatility paradox
- Financial innovation  less stable economy
Full model

Experts

- Output: \( y_t = ak_t \)
- Consumption rate: \( c_t \)
- Investment rate: \( l_t \)
  \[
  \frac{dk_t}{k_t} = (\Phi(l_t) - \delta)dt + \sigma dZ_t
  \]

- \( E_0[\int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt] \)

- Can issue
  - Risk-free debt
  - Equity, but most hold \( \chi_t \geq \chi \)

Households

- Output: \( y_t = ak_t \)
- Consumption rate: \( c_t \)
- Investment rate: \( l_t \)
  \[
  \frac{dk_t}{k_t} = (\Phi(l_t) - \delta)dt + \sigma dZ_t
  \]

- \( E_0[\int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt] \)

- \( a \geq a \)
- \( \delta \leq \delta \)
Experts

- Experts must hold fraction $\chi_t \geq \chi$
Solution steps

1. Postulate endogenous processes
   • \( \frac{dq_t}{q_t} =, \frac{d\theta_t}{\theta_t} =.., \frac{d\theta_t}{\theta_t} = \mu_t \, dt + \sigma_t \, dZ_t \)

2. Equilibrium conditions
   • Agents’ optimization
     ▪ Internal investment (new capital formation)
     ▪ Optimal portfolio choice with equity issuance
     ▪ Optimal consumption
   • Market clearing conditions

3. Law of motion of state variable
   • wealth (share) distribution \( \eta_t \)

4. Express in ODEs of state variable
2. Optimal portfolio condition

- Without equity issuance

\[
\frac{a-l_t + \Phi(l_t) - \delta + \mu_t^q + \sigma \sigma_t^q - r_t}{\sigma + \sigma^q} = -\sigma_t \theta
\]

\[
\frac{a-l_t + \Phi(l_t) - \delta + \mu_t^q + \sigma \sigma_t^q - r_t}{\sigma + \sigma^q} \leq -\sigma_t \theta \quad \text{with equality if } \psi_t < 1
\]
2. Optimal portfolio condition

Without equity issuance

\[
\frac{a - \ell_t + \Phi(\ell_t) - \delta + \mu_t^q + \sigma \sigma_t^q - r_t}{\sigma + \sigma^q} = -\sigma_t^\theta \chi_t (-\sigma_t^\theta) + (1 - \chi_t)(-\sigma_t^\theta)
\]

\[
\frac{a - \ell_t + \Phi(\ell_t) - \delta + \mu_t^q + \sigma \sigma_t^q - r_t}{\sigma + \sigma^q} \leq -\sigma_t^\theta \quad \text{with equality if } \psi_t < 1
\]

If experts require higher returns than HH

- if \(-\sigma^\theta > -\sigma_t^\theta \Rightarrow \chi_t = \chi\)
- Otherwise \(-\sigma^\theta = -\sigma_t^\theta\)

\[
\frac{(a-a)/q_t}{\sigma + \sigma_t^q} \geq \chi \quad \text{with equality if } \psi_t < 1
\]
3. Law of motion of $\eta_t$

\[
\frac{dN_t}{N_t} = r_t dt + \frac{\chi_t \psi_t (\sigma + \sigma_t^q)}{\eta_t} \left( -\sigma_t^\theta \right) dt \\
+ \frac{\chi_t \psi_t (\sigma + \sigma_t^q)}{\eta_t^2} dZ_t - \frac{C_t}{N_t} dt
\]

- Use Ito ratio rule for $\eta_t = N_t / (q_t K_t)$

\[
\frac{d\eta_t}{\eta_t} = ..
\]
4. Express in functions $q(\eta), \theta(\eta), \psi(\eta), \chi(\eta)$

- Convert equilibrium conditions and law of motion
- Replace terms $\mu^q_t, \mu^\theta_t, \sigma^q_t, \sigma^\theta_t, \ldots$ with expressions containing derivatives of $q$ and $\theta$ – using Ito’s lemma
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- A simple example: Leland (1994)
  - $dV_t = rV_t dt + \sigma V_t dZ_t$ (under Q) default at $V_t = V_B$ to $\alpha V_B$
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A simple example: Leland (1994)

- \( dV_t = rV_t \, dt + \sigma V_t \, dZ_t \) (under Q) default at \( V_t = V_B \) to \( \alpha V_B \)
  1. Postulate \( dE_t = \mu_t^E E_t \, dt + \sigma_t^E E_t \, dZ_t \)
  2. Equilibrium condition: \( r = \mu_t^E - \frac{c}{E_t} \)
4. Express in functions $q(\eta), \theta(\eta), \psi(\eta), \chi(\eta)$

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3. Ito lemma on $E(V)$: $\mu^E_t E_t = E'(V_t) rV_t + \frac{1}{2} \sigma^2 V_t^2 E''(V_t)$
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  1. Postulate $dE_t = \mu_t^E E_t dt + \sigma_t^E E_t dZ_t$
  2. Equilibrium condition: $r = \mu_t^E - \frac{c}{E_t}$
  - Ito lemma on $E(V)$: $\mu_t^E E_t = E'(V_t) rV_t + \frac{1}{2} \sigma^2 V_t^2 E''(V_t)$

- New equilibrium condition: $r = \frac{E'(V_t) rV + \frac{1}{2} \sigma^2 V_t^2 E''(V_t)}{E(V)} - \frac{c}{E(V)}$
  - Two boundary conditions
    1. $E(V_B) = 0$
    2. $V - E(V) \to \frac{c}{r}$ as $V \to \infty$
Amplification – closed form

\[ \sigma_t^n = \frac{\frac{\chi_t \psi_t}{\eta_t} - 1}{1 - \left[ \frac{\chi_t \psi_t}{\eta_t} - 1 \right] \frac{q'(\eta_t)}{q(\eta_t)/\eta_t}} \]

- Leverage effect \( \frac{\chi_t \psi_t}{\eta_t} - 1 \)
- Loss spiral \( 1/\left\{ 1 - \left[ \frac{\chi_t \psi_t}{\eta_t} - 1 \right] \frac{q'(\eta_t)}{q(\eta_t)/\eta_t} \right\} \) (infinite sum)

- Technological illiquidity \((\kappa, \delta)\) ⇒ market illiquidity \(q'(\eta)\)
  - (dis)investment adjustment cost
5. Solving system of ODE numerically

- Matlab ODE solver, ode45

- Boundary conditions
  - \( \theta(0) = M \) for large constant \( M \)
  - \( \theta'(\eta) \)
  - \( q(0) = \) (closed form for log utility and log \( \Phi \))
Monetary Models

- “Money models” **without intermediaries**
  - Store of value: Money pays no dividend and is a bubble

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- **With intermediaries/inside money**
  - “Money view” (Friedman & Schwartz) vs. “Credit view” (Tobin)
Monetary Models – The I Theory of Money

- **Step 1:** Postulate process for value of money $p_t K_t$
  - \[
  \frac{dp_t}{p_t} = \mu^p_t \, dt + \sigma^p_t \, dZ_t \quad \text{(money + bond)}
  \]
  - \[
  dB_t / B_t = \mu^B_t \, dt + \sigma^B_t \, dZ_t \quad \text{(part due to consul bond)}
  \]
Conclusion

- Manual for continuous time macro-finance models
  - 4 step approach
- More tractable: explicit amplification terms
- Volatility dynamics characterization
  - Precautionary motive
  - Endogenous fat tails, crisis probability
- Undercapitalized sectors, liquidity mismatch, fire-sales, equity issuance cycles, fat tails,
- Revival of “Money and Banking”
  - The I Theory of Money with short-term money and long-term bond