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Macro, Money and Finance: A Continuous Time Approach

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- Price stability
  Monetary policy

- Financial stability
  Macroprudential policy

- Fiscal debt sustainability
  Fiscal policy

- Short-term interest
- Policy rule (terms structure)

inter-action

- Reserve requirements
- Capital/liquidity requirements
- Collateral policy
  Margins/haircuts
- Capital controls

inter-action

Macro, Money and Finance

- Endogenous level
  - Persistence & amplification
  - “Net worth trap”
- Endogenous risk dynamics
  - Tail risk
  - Crisis probability
  - “Volatility Paradox”
- Illiquidity and liquidity mismatch
  - Undercapitalized sectors
  - Time varying risk premia (dynamics)
  - External funding premium
- Value of money
- Welfare
  - Interaction: regulatory, monetary and other policies

History: Macro & Finance

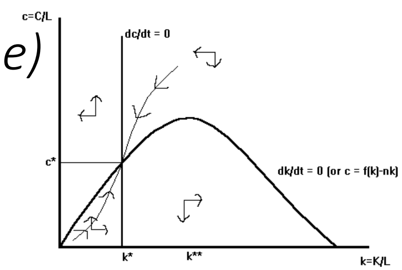
timeline

- *Verbal Reasoning* *(qualitative)*

Fisher, Keynes, …

Macro

- Growth theory
  - *Dynamic (cts. time)*
  - *Determinisitc*

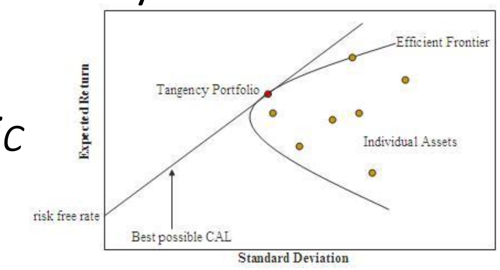


- Introduce stochastic
  - *Discrete time*
    - Brock-Mirman
    - Kydland-Prescott
    - DSGE models

Stokey-Lucas

Finance

- Portfolio theory
  - *Static*
  - *Stochastic*

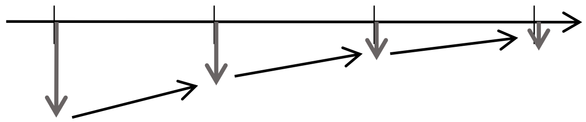


- Introduce dynamics
  - *Continuous time*
    - Options — Black Scholes
    - Term structure — CIR
    - Agency theory — Sannikov

- Cts. time macro with financial frictions

Amplification & Persistence

- Bernanke & Gertler (1989), Carlstrom & Fuerst (1997)
  - Perfect (technological) liquidity, but persistence
  - Bad shocks erode net worth, cut back on investments, leading to low productivity & low net worth of in the next period



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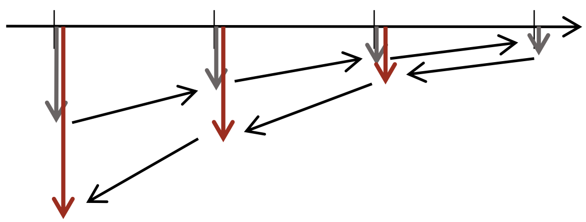

- Kiyotaki & Moore (1997), BGG (1999)
  - Technological/market illiquidity
  - KM: Leverage bounded by margins; BGG: Verification cost (CSV)
  - Stronger amplification effects through prices (low net worth reduces leveraged institutions' demand for assets, lowering prices and further depressing net worth)

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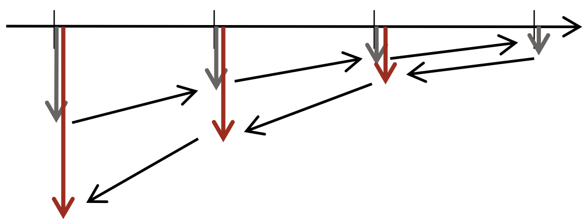

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- "once and for all shock"
  - no volatility dynamics

Impulse response vs. Volatility dynamics

- “once and for all shock”
  = no uncertainty about length of slump
- Sequence of adverse shock

Why continuous time modeling?

- Characterization for volatility and amplification
  - Discrete: only impulse response functions
    - Only for shocks starting at the steady state
    - Only expected path – fan charts help somewhat
- More analytical steps
  - Return equations
    - Next instant returns are essentially log normal
      (easy to take expectations)
  - Explicit net worth and state variable dynamics
    - Continuous: only slope of price function determines amplification
    - Discrete: need whole price function (as jump size can vary)
- Numerically simple – solve differential equations
- Discrete: IES/RA within period $= \infty$, across periods $1/\gamma$

Cts. time: special features of diffusions

- Continuous path – fast enough deleveraging
  - Never jumps over a specific point, e.g. insolvency point
- Implicit assumption: can react to small price changes
  - Can continuously delever as wealth goes down
  - Makes them more bold ex-ante

Recent macro literature (in cts time)

- Core
  - BrunSan (2014), Basak & Cuoco (1998) He & Krishnamurthy (2012,13), DiTella (2013), Isohätälä et al. (2014)
- Intermediation/shadow banking
  - Phelan (2014), Adrian & Boyarchenko (2012,13), Huang (2014), Moreira & Savov (2014), Klimenko & Rochet (2015)
- Quantification
  - He & Krishnamurthy (2014), Mittnik & Semmler (2013)
- International
  - BruSan (2015), Maggiori (2013)
- Monetary
  - "The I Theory of Money" (2012), Drechsler et al. (2014)
- ….

Financial frictions

- Costly state verification (BGG)

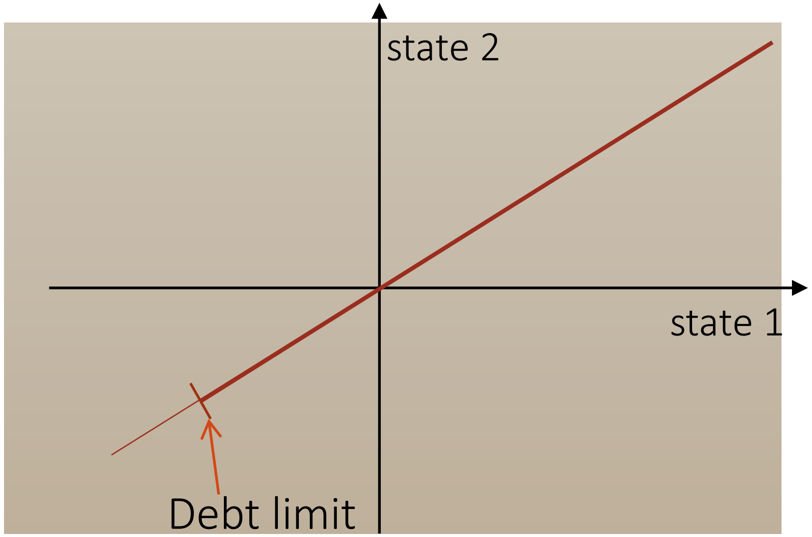
- Leverage constraints
  - Exogenous limit (Bewley/Ayagari)

  - Collateral constraints
    - Next period's price (KM)

      $$Rb_t \leq q_{t+1}k_t$$

    - Next periods volatility (VaR)
    - Current price

- Incomplete markets
  - Endogenous leverage



can depend on prices/volatility

Roadmap

- Why continuous time?
- Literature

- Simple model
  - With undesirable features
  - Add portfolio choice with general utility function
- Full model
  - With all desired features
  - Add equity issuance

A simple model

Basak & Cuoco (1998)

Experts

- Output: $y_t = ak_t$
- Consumption rate: c_t
- Investment rate: ι_t

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \sigma dZ_t$$

- $E_0[\int_0^\infty e^{-\rho t} \log(c_t)\, dt]$

- Can only issue risk-free debt

Households

- No output: $\underline{a} = 0$
- Consumption rate: $\underline{c}_t$

- $E_0[\int_0^\infty e^{-\rho t} \log(\underline{c}_t)\, dt]$

Equilibrium

- An equilibrium consists of functions that for each history of macro shocks $\{Z_s, s \in [0,t]\}$ specify
  - q_t the price of capital
  - $k_t, \underline{k}_t = 0$ capital holdings
  - $c_t \geq 0, \underline{c}_t = 0$ consumption of representative expert and households
  - $\iota_t, \underline{\iota}_t = 0$ rate of internal investment, per unit of capital
  - r the risk-free rate
- such that
  - intermediaries and households maximize their utility, taking prices q_t as given and
  - markets for capital and consumption goods clear

Equilibrium

- Equilibrium is a map

Histories of shocks → prices, allocations

$\{Z_s, s \leq t\}$ ⇢ $q_t, \psi_t, \iota_t, \underline{\iota}, c_t, \underline{c}_t$

wealth distribution

$$\eta_t = \frac{N_t}{q_t K_t} \in (0,1)$$

experts' wealth share

- Experts, HH solve optimal investment, portfolio, consumption
- Markets clear

Solution steps

1. Postulate endogenous processes
   - $dq_t/q_t = \mu_t^q dt + \sigma_t^q dZ_t$
     - Returns from holding capital
2. Equilibrium conditions
   - Agents' optimization
     - Internal investment (new capital formation)
     - Optimal portfolio choice
     - Optimal consumption
   - Market clearing conditions
3. Law of motion of state variable
   - wealth (share) distribution η_t
4. Express in ODEs of state variable

1. Postulate endogenous process

- Postulate
  - $dq_t/q_t = \mu_t^q dt + \sigma_t^q dZ_t$

Recall

$dk_t/k_t = (\Phi(\iota_t) - \delta) + \sigma dZ_t$

- Return on capital
  - $dr_t^k = \underbrace{\frac{a-\iota_t}{q_t}}_{\substack{dividend\\yield}} dt + \underbrace{\frac{d(k_t q_t)}{k_t q_t}}_{\substack{capital\\gains}}$

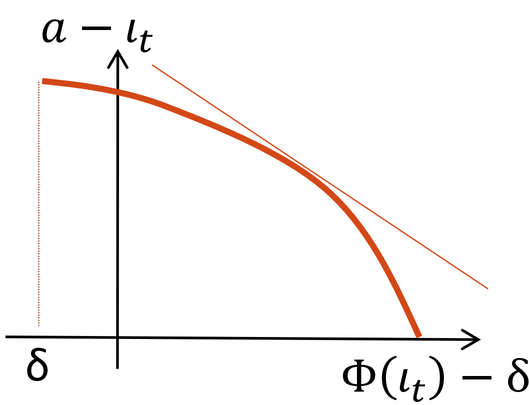
  - $\frac{d(k_t q_t)}{k_t q_t} = \left(\Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)dt + \left(\sigma + \sigma_t^q\right)dZ_t$

    by Ito's product rule

- In this simple model it will turn out that q is constant, i.e. $\mu_t^q = \sigma_t^q = \mathbf{0}$.

2. Equilibrium optimality conditions

a. Investment rate (capital formation)



- Static problem

$$\max_{\iota} \Phi(\iota_t) - \iota_t / q_t$$

- FOC: $\Phi'(\iota_t) = \frac{1}{q_t}$ (marginal Tobin's q)

b. Consumption choice

- $c_t = \rho N_t$ due to log utility

c. Portfolio choice

- Volatility of wealth = Sharpe ratio of risky investment

2. Equilibrium market clearing conditions

- Goods market
  price of capital

$$\rho q_t K_t = \big(a - \iota_t(q_t)\big) K_t$$

  - $q_t = q = \cdots$
  - Special case: $\Phi(\iota) = \frac{\log(\kappa\iota+1)}{\kappa}$ $\quad \iota = \frac{(q-1)}{\kappa}$

$$q = \frac{a+1/\kappa}{r+1/\kappa}$$

- Risk free rate
  - $dr_t^k = \underbrace{\frac{a-\iota_t}{q_t}}_{\rho, dividend\ yield} + \underbrace{(\Phi(\iota_t)-\delta)dt+\sigma dZ_t}_{capital\ gains}$
  - Sharpe ratio: $\frac{\rho+\Phi(\iota)-\delta-r_t}{\sigma}$
  - Volatility of net worth: $\frac{q_t K_t}{N_t}\sigma = \frac{\sigma}{\eta_t}$
  - Sharpe ratio = volatility of N_t $\quad r_t = \rho + \Phi(\iota) - \delta - \frac{\sigma^2}{\eta_t}$

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3. Law of motion of η_t

- $\frac{dN_t}{N_t} = r_t dt + \underbrace{\frac{\sigma}{\eta_t}}_{risk} * \underbrace{\frac{\sigma}{\eta_t}}_{Sharpe} dt + \frac{\sigma}{\eta_t} dZ_t - \underbrace{\rho dt}_{consumption}$
- $\frac{d(q_t K_t)}{q_t K_t} = ..$
- Use Ito ratio rule for $\eta_t = N_t/(q_t K_t)$

$$\frac{d\eta_t}{\eta_t} = \frac{(1-\eta_t)^2}{\eta_t^2}\sigma^2 dt + (1-\eta_t)\sigma dZ_t$$

Observations

- $\frac{d\eta_t}{\eta_t} = \frac{(1-\eta_t)^2}{\eta_t^2}\sigma^2 dt + (1-\eta_t)\sigma dZ_t$
  - Wealth share η moves with macro shock dZ_t
  - In the long run experts "save their way out", $\eta \to 1$

- Sharpe ratio $\frac{\rho+\Phi(\iota)-\delta-r_t}{\sigma}$
  - Increases as η goes down, (to ∞ as $\eta \to 0$)
  - Achieved through a lower risk free rate

- q is constant
  - No endogenous risk
  - No amplification
  - No volatility effects

Generalizing preference: portfolio choice

1. Also postulate process for marginal utility
   $d\theta_t/\theta_t = \mu_t^\theta dt + \sigma_t^\theta dZ_t$ SDF: $e^{\rho s}\theta_{t+s}/\theta_t$
2. Portfolio choice: Optimality condition
   - For asset A with payoff process $dr_t^A = \mu_t^A dt + \sigma_t^A dZ_t$

$$0 = \mu_t^\theta - \rho + \mu_t^A + \sigma_t^A \sigma_t^\theta$$

   - Intuition:
     i. Discrete time analog: Take log of $1 = E_t\left[SDF_{t,t+s}\left(R_{t,t+s}\right)\right]$
     ii. Consider wealth n_t invested in A, so that $dn_t/n_t = dr_t^A$
     $n_{t+s}e^{-\rho s}\frac{\theta_{t+s}}{\theta_t}$ is a martingale

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   - For risk free asset

$$0 = \mu_t^\theta - \rho + r$$

   - Sharpe ratio

$$\frac{\mu_t^A - r_t}{\sigma_t^A} = -\sigma_t^\theta$$

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Example 1: $u(c) = \log(c)$

$\theta_t = \frac{1}{c_t} = \frac{1}{\rho n_t} \Rightarrow \sigma_t^\theta = -\sigma_t^n$

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$$0 = \mu_t^\theta - \rho + r$$

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$\theta_t = \frac{1}{c_t} = \frac{1}{\rho n_t} \quad \Rightarrow \quad \sigma_t^\theta = -\sigma_t^n$

Example 2: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$

$.. \Rightarrow \quad \sigma_t^\theta = -\gamma \sigma_t^c$

Desired model properties

- Normal regime: stable around steady state
  - Experts are adequately capitalized
  - Experts can absorb macro shock
- Net worth trap look at stationary distribution
- Endogenous risk
  - Fat tails
  - Assets are more correlated
  - SDF vs. cash-flow news
- Volatility paradox
- Financial innovation less stable economy

Full model

Experts

- Output: $y_t = ak_t$
- Consumption rate: c_t
- Investment rate: ι_t

$$\frac{dk_t}{k_t}=(\Phi(\iota_t)-\delta)dt+\sigma dZ_t$$

- $E_0[\int_0^\infty e^{-\rho t}\frac{c_t^{1-\gamma}}{1-\gamma}dt]$

- Can issue
  - Risk-free debt
  - Equity, but most hold $\chi_t \geq \underline{\chi}$

$a \geq \underline{a}$

$\delta \leq \underline{\delta}$

$\rho \geq \underline{\rho}$

Households

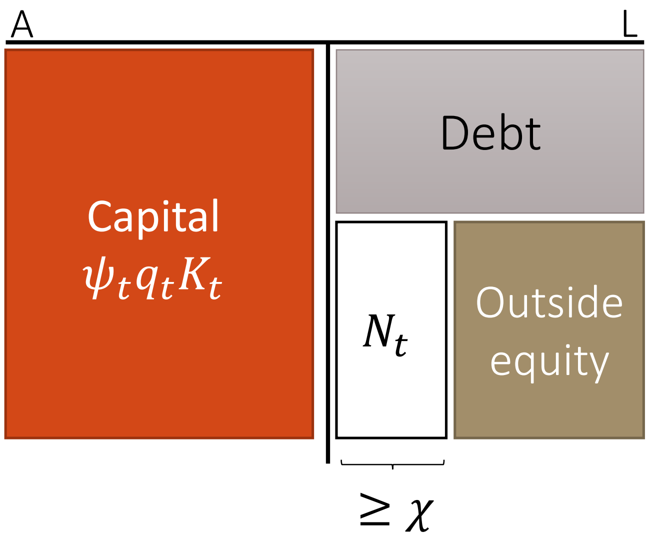
- Output: $\underline{y}_t = \underline{a}\underline{k}_t$
- Consumption rate: $\underline{c}_t$
- Investment rate: ι_t

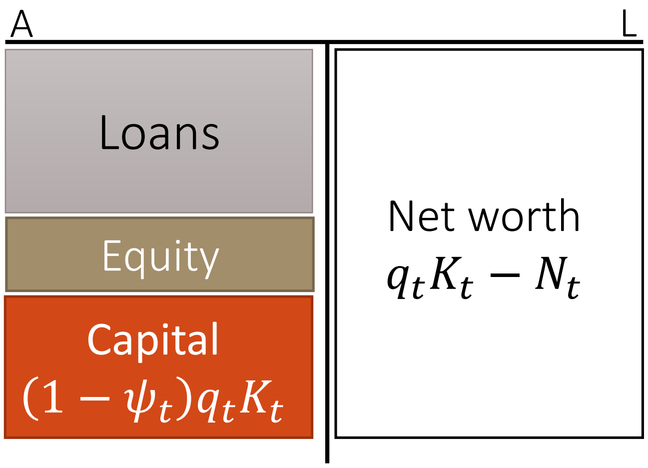
$$\frac{dk_t}{k_t}=(\Phi(\underline{\iota}_t)-\underline{\delta})dt+\sigma dZ_t$$

- $E_0[\int_0^\infty e^{-\underline{\rho} t}\frac{c_t^{1-\gamma}}{1-\gamma}dt]$

Experts

Households





- Experts must hold fraction $\chi_t \geq \underline{\chi}$

Solution steps

1. Postulate endogenous processes
   - $dq_t/q_t =,\quad d\theta_t/\theta_t =..,\qquad d\underline{\theta}_t/\underline{\theta}_t = \mu_t^{\underline{\theta}} dt + \sigma_t^{\underline{\theta}} dZ_t$
2. Equilibrium conditions
   - Agents' optimization
     - Internal investment (new capital formation)
     - Optimal portfolio choice with equity issuance
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4. Express in ODEs of state variable

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2. Optimal portfolio condition

- Without equity issuance

$$\frac{\frac{a-\iota_t}{q_t}+\Phi(\iota_t)-\delta+\mu_t^q+\sigma\sigma_t^q-r_t}{\sigma+\sigma^q} = -\sigma_t^\theta$$

$$\frac{\frac{\underline{a}-\underline{\iota}_t}{q_t}+\Phi(\underline{\iota}_t)-\delta+\mu_t^q+\sigma\sigma_t^q-r_t}{\sigma+\sigma^q} \leq -\sigma_t^{\underline{\theta}} \quad \text{with equality if } \psi_t < 1$$

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- Without equity issuance

$$\frac{\frac{a-\iota_t}{q_t}+\Phi(\iota_t)-\delta+\mu_t^q+\sigma\sigma_t^q-r_t}{\sigma+\sigma^q} = -\sigma_t^\theta \; \chi_t(-\sigma_t^\theta) + (1-\chi_t)(-\sigma_t^\theta)$$

$$\frac{\frac{\underline{a}-\underline{\iota}_t}{q_t}+\Phi(\underline{\iota}_t)-\delta+\mu_t^q+\sigma\sigma_t^q-r_t}{\sigma+\sigma^q} \leq -\sigma_t^\theta \quad \text{with equality if } \psi_t < 1$$

- If experts require higher returns than HH
  - if $-\sigma^\theta > -\sigma_t^\theta \Rightarrow \chi_t = \underline{\chi}$
  - Otherwise $-\sigma^\theta = -\sigma_t^\theta$
- $\frac{(a-\underline{a})/q_t}{\sigma+\sigma_t^q} \geq \underline{\chi}$ with equality if $\psi_t < 1$

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3. Law of motion of η_t

$$\frac{dN_t}{N_t} = r_t dt + \underbrace{\frac{\chi_t \psi_t}{\eta_t}(\sigma + \sigma_t^q)}_{risk} \underbrace{(-\sigma_t^\theta)}_{\substack{risk \\ premium}} dt$$

$$+ \frac{\chi_t \psi_t}{\eta_t}(\sigma + \sigma_t^q) dZ_t - \frac{C_t}{N_t} dt$$

- Use Ito ratio rule for $\eta_t = N_t/(q_t K_t)$

$$\frac{d\eta_t}{\eta_t} = ..$$

4. Express in functions $q(\eta), \theta(\eta), \psi(\eta), \chi(\eta)$

- Convert equilibrium conditions and law of motion
- Replace terms $\mu_t^q, \mu_t^\theta, \sigma_t^q, \sigma_t^\theta, \ldots$ with expressions containing derivatives of q and θ – using Ito's lemma

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- A simple example: Leland (1994)
  - $dV_t = rV_t dt + \sigma V_t dZ_t$ (under Q) default at $V_t = V_B$ to αV_B

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  1. Postulate $dE_t = \mu_t^E E_t dt + \sigma_t^E E_t dZ_t$
  2. Equilibrium condition: $r = \mu_t^E - {}^{C}/_{E_t}$

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  - Ito lemma on $E(V)$: $\mu_t^E E_t = E'(V_t) r V_t + \frac{1}{2}\sigma^2 V_t^2 E''(V_t)$

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  1. Postulate $dE_t = \mu_t^E E_t dt + \sigma_t^E E_t dZ_t$
  2. Equilibrium condition: $r = \mu_t^E - {}^C/_{E_t}$
  - Ito lemma on $E(V)$: $\mu_t^E E_t = E'(V_t) r V_t + \frac{1}{2}\sigma^2 V_t^2 E''(V_t)$
  2. New equilibrium condition: $r = \frac{E'(V) rV + \frac{1}{2}\sigma^2 V_t^2 E''(V_t)}{E(V)} - \frac{C}{E(V)}$
     - Two boundary conditions
     1. $E(V_B) = 0$
     2. $V - E(V) \to \frac{C}{r}$ as $V \to \infty$

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Amplification – closed form

$$\sigma_t^\eta = \frac{\frac{\chi_t \psi_t}{\eta_t} - 1}{1 - \left[\frac{\chi_t \psi_t}{\eta_t} - 1\right] \frac{q'(\eta_t)}{q(\eta_t)/\eta_t}} \sigma$$

"leverage"

Market illiquidity (price impact elasticity)

- Leverage effect $\frac{\chi_t \psi_t}{\eta_t} - 1$
- Loss spiral $1/\{1 - [\frac{\chi_t \psi_t}{\eta_t} - 1]\frac{q'(\eta_t)}{q(\eta_t)/\eta_t}\}$ (infinite sum)
- Technological illiquidity $(\kappa, \delta) \Rightarrow$ market illiquidity $q'(\eta)$
  - (dis)investment adjustment cost

5. Solving system of ODE numerically

- Matlab ODE solver, ode45

- Boundary conditions
  - $\theta(0) = M$ for large constant M
  - $\theta'(\eta)$
  - $q(0) =$ (closed form for log utility and log Φ)

Monetary Models

- "Money models" <u>without intermediaries</u>
  - Store of value: Money pays no dividend and is a bubble

\Friction	OLG	Incomplete Markets + idiosyncratic risk	
Risk	deterministic	endowment risk borrowing constraint	investment risk
Only money	Samuelson	Bewley	
With capital	Diamond	Aiyagari, Krusell-Smith	Basic "I Theory"

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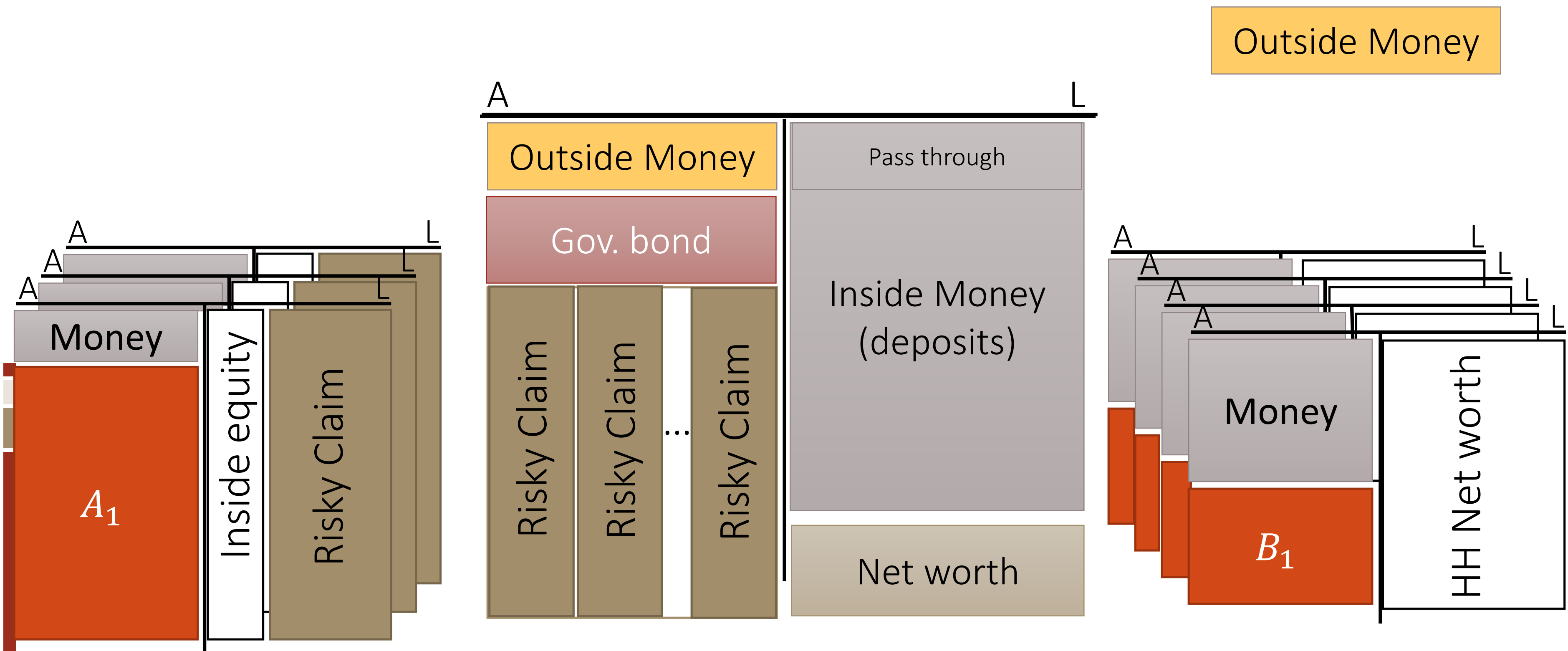
- <u>With intermediaries/inside money</u>
  - "Money view" (Friedman & Schwartz) vs. "Credit view"(Tobin)

Monetary Models – The I Theory of Money

- Step 1: Postulate process for value of money $p_t K_t$
  - $\frac{dp_t}{p_t} = \mu_t^p dt + \boldsymbol{\sigma}_t^p d\boldsymbol{Z}_t$ vectors (money + bond)
  - $dB_t/B_t = \mu_t^B dt + \boldsymbol{\sigma}_t^B d\boldsymbol{Z}_t$ (part due to consul bond)



Brunnermeier & Sannikov

Conclusion

- Manual for continuous time macro-finance models
  - 4 step approach
- More tractable: explicit amplification terms
- Volatility dynamics characterization
  - Precautionary motive
  - Endogenous fat tails, crisis probability
- Undercapitalized sectors, liquidity mismatch, fire-sales, equity issuance cycles, fat tails,
- Revival of "Money and Banking"
  - The I Theory of Money
    with short-term money and long-term bond